Polarization Plateau in Atomic Fermi Gas Loaded on Triangular Optical Lattice

M. Okumura,[1,2,*] S. Yamada,[1,2,†] M. Machida,[1,2,‡] and T. Sakai[3,4,§]

[1] *CCSE, Japan Atomic Energy Agency, 6–9–3 Higashi-Ueno, Taito-ku, Tokyo 110–0015, Japan*
[2] *CREST (JST), 4–1–8 Honcho, Kawaguchi, Saitama 332–0012, Japan*
[3] *Japan Atomic Energy Agency, SPring-8, Sayo, Hyogo 679–5148, Japan*
[4] *Department of Material Science, University of Hyogo, Kamigori, Hyogo 678–1297, Japan*

(Dated: February 19, 2009)

In order to demonstrate that atomic Fermi gas is a good experimental reality in studying unsolved problems in frustrated interacting-spin systems, we numerically examine the Mott core state emerged by loading two-component atomic Fermi gases on triangular optical lattices. Consequently, we find that plateau like structures are observable in the Mott core polarization as a function of the population imbalance. These plateau states are caused by a flexibility that the surrounding metallic region absorbs the excess imbalance to keep the plateau states inside the Mott core. We also find spin patterns peculiar to the plateau states inside the Mott core.

PACS numbers: 67.85.Lm, 03.75.Ss, 71.10.Fd, 75.40.Mg

Interplay between geometrical frustration and quantum fluctuation in low-dimensional condensed matters allows non-trivial ground state. One of the typical example is the ground state of the isotropic triangular $S = 1/2$ antiferromagnetic Heisenberg and *XXZ* models. Anderson and Fazekas proposed that the resonating valence bond state is a candidate of the ground state [1]. Contrary to the conjecture, numerical calculations suggested 120° Néel-ordered ground state [2]. On the other hand, several experiments showed that the ground state is a spin liquid [3, 4, 5, 6, 7]. But, its excitation feature differs even in experiments using the same materials. One claimed gapless [6], and another gapful [7]. These controversies require more systematic and precise experiments.

Another typical example is 1/3 magnetization plateau state in triangular antiferromagnetic Heisenberg and *XXZ* models under the magnetic field. Miyashita suggested the existence based on classical analysis [8], and both numerical calculations [9, 10, 11, 12] and experiments [13] confirmed it. In addition, Oshikawa, Yamanaka, and Affleck (OYA) advocated a related quantization condition for systems with periodic boundary condition given by $q(\mathcal{S} - m) =$ integer, where q is an integer being the size ratio of the ground-state unit-cell to the original one and $\mathcal{S}$ and m are the spin quantum number and the average magnetization per unit cell of the system, respectively [14]. The OYA quantization condition predicts how a translational symmetry is broken and how a magnetization value causing the plateau is given. In fact, using the density-matrix renormalization group (DMRG) method [15, 16], Okunishi and Tonegawa actually confirmed a spin distribution, in which a translational symmetry is broken, in a zigzag chain model [11].

Presently, it is known that 1/3 plateau in the *XXZ* model exists in both the quantum model and the classical counterpart while 1/3 plateau in the Heisenberg model and 2/3 plateau in both models arises from pure quantum origin [8, 9, 11]. Moreover, there exists a critical ratio of the distortion of the triangular lattice in the Heisenberg model and in the *XXZ* model for existence of the plateau [9, 10, 11, 12, 17]. Thus, a remaining issue is to clear how the plateau states appear and disappear in parameter variations of frustrated models. This clearly requires a quite controllable experiment for the distortion.

In this paper, we suggest that atomic Fermi gas loaded on optical lattice (FGOL) [18] can offer a crucial stage on such controversial topics. Our propose is based on the following features peculiar to FGOL. The repulsively interacting two-component FGOL creates the so-called Mott core in the central region due to the existence of both a repulsive interaction and a harmonic trap brought about by the laser intensity profile [19, 20] as schematically shown in Fig. 1(a) and 1(b). The Mott core state, which was recently confirmed experimentally [21, 22], is mapped onto $S = 1/2$ Heisenberg spin model, since the spin degree of freedom survives solely. In a triangular optical lattice, which is easily accessible in FGOL, the distortion is almost freely controllable by adjusting the angle between the counter laser beams, and the arbitrary anisotropy variation is also achievable by producing spin-state dependent difference on the atom hopping [23]. Moreover, the application of the magnetic field is mimicked by changing the population imbalance [24]. Thus, FGOL is found to be a quite flexible experimental reality to study quantum frustrated systems systematically. However, we note that there is only a big difference between condensed-matter spin systems and FGOL. The Mott core as stage of spin models is surrounded by a metallic periphery having a role of "environment" to the spin system. Then, it is not obvious whether or not the feature is insignificant in studying the above critical issues. Therefore, main aims of this paper are to perform direct simulations of the trapped frustrated FGOL and

*Electronic address: okumura.masahiko@jaea.go.jp
†Electronic address: yamada.susumu@jaea.go.jp
‡Electronic address: machida.masahiko@jaea.go.jp
§Electronic address: sakai@spring8.or.jp

to actually confirm the advantage of use of FGOL. In this paper, we concentrate on 1/3 plateau and related states as a trial problem on the trapped frustrated FGOL. Other issues are now under investigations.

In FGOL, the magnetic field and the magnetization are replaced by the imbalance ratio and the polarization, respectively. We solve a triangular Hubbard model with a harmonic trap potential for x-direction [Fig. 1(a) and 1(b)] given by

$$H = -t \sum_{\sigma,\langle \boldsymbol{i},\boldsymbol{j}\rangle} c^{\dagger}_{\sigma\boldsymbol{i}} c_{\sigma\boldsymbol{j}} + V \sum_{\sigma,\boldsymbol{i}} \left[(\boldsymbol{r}_{\boldsymbol{i}} - \boldsymbol{r}_{\mathrm{c}}) \cdot \boldsymbol{e}_x\right]^2 n_{\sigma\boldsymbol{i}} + U \sum_{\boldsymbol{i}} n_{\uparrow\boldsymbol{i}} n_{\downarrow\boldsymbol{i}} \,, \tag{1}$$

where $\langle \boldsymbol{i},\boldsymbol{j}\rangle$ refers to the nearest neighbors, $\sigma =\uparrow$ and $\downarrow$, $\boldsymbol{r}_{\boldsymbol{i}}$ and $\boldsymbol{r}_{\mathrm{c}}$ are the position vector of the $\boldsymbol{i}$-th site and the center of the system, $\boldsymbol{e}_x$ is a unit vector for x-direction, t is the hopping parameter, U is the on-site repulsion, V is the harmonic potential strength, $c_{\sigma\boldsymbol{i}}$ ($c^{\dagger}_{\sigma\boldsymbol{i}}$) is the annihilation- (creation-) operator and $n_{\sigma\boldsymbol{i}} (\equiv c^{\dagger}_{\sigma\boldsymbol{i}} c_{\sigma\boldsymbol{i}})$ is the site density operator. We calculate the imbalance ratio $p (\equiv \sum_{\boldsymbol{i}} N_{-,\boldsymbol{i}}/N_{+,\boldsymbol{i}})$ vs. the normalized polarization on the emergent Mott core, $M = \sum_{\boldsymbol{i}\in\mathcal{M}} N_{-,\boldsymbol{i}}/N_{+,\boldsymbol{i}}$ [see Fig. 1(c) and 1(d)], where $N_{\pm,\boldsymbol{i}} \equiv n_{\uparrow\boldsymbol{i}} \pm n_{\downarrow\boldsymbol{i}}$. The calculations are made by using DMRG method [15, 16] on 3×34-sites triangular ladder with 60 fermions with $U/t = 10$ and $V/t = 0.07$. Our DMRG is directly extended to ladder systems by parallelizing the superblock matrix diagonalization. See Ref. [25] for more details of the parallelization and related techniques. We confirm the precision of DMRG results in short 3-leg ladders by comparing those of the exact diagonalization, and check a dependence of results on the number of states kept in long ladders to obtain reliable results. Then, the number of states kept is changed from 300 to 700 according to the convergence check. 700 is enough for every case.

Let us present DMRG calculation results. Figure 2 shows the spin imbalance ratio p vs. the polarization on the Mott core M. In Fig. 2, one finds three characteristic features. The first is a plateau like structure seen around $p = 1/3$, the second is a kink around $p = 2/3$, and the third is another plateau one indicating fully magnetized before $p = 1$. According to the OYA quantization condition, $\mathcal{S} = 3/2$ is derived in 3-leg ladder. Consequently, we have $m = 1/2$ ($q = 1$) and $m = 1$ ($q = 2$) in 1/3 and 2/3 plateaus, respectively. These conditions then predict for periodic systems that the number of sites in the unit cells of the ground state are given by multiples of 3 and 6, respectively in 1/3 and 2/3 plateaus, respectively. The spin distribution in the 1/3 polarization plateau is compared with the OYA prediction in 1/3 magnetization plateau. The 2/3 polarization kink is not a full plateau, but the observed spin structure is expected to be relevant to the OYA quantization condition in 2/3 magnetization plateau.

Now, let us examine spin distributions on the points indicated in Fig. 2. The first focus is 1/3 plateau observed from $p = 0.27$ to $p = 0.33$, in which we find two characteristic patterns. Figure 3(a) shows a spin distribution pattern at $p = 0.30$, and Fig. 3(b) another one at $p = 0.33$. The pattern at $p = 0.27$ is almost equivalent to Fig. 3(b) ($p = 0.33$). From these patterns, it is found that both are characterized by the 3-sites periodicity along the ladder direction. In Fig. 3(a), *up-zero-zero* is observed along the direction, and *up-up-down* [8, 9, 10, 11, 12, 13, 17] in Fig. 3(b). These results indicate that the polarization value is always 1/3 within the Mott core although the population imbalance ratio is different. It is clearly found that the difference of the population imbalance is absorbed in the metallic periphery, i.e., “environment” and 1/3 magnetization is kept inside the Mott core. This robustness of the plateau state is crucial for experimental

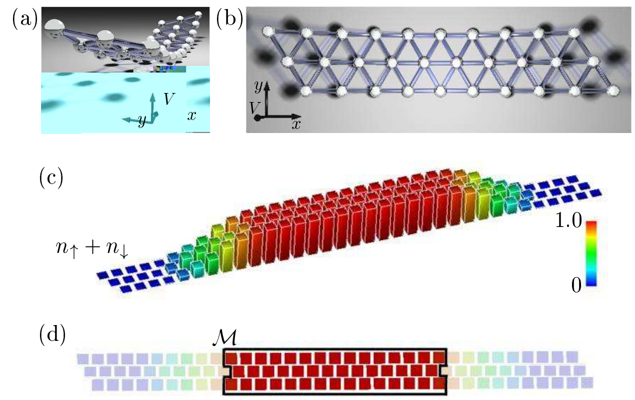

FIG. 1: (a), (b) Schematic figures of the system considered in this paper. The x- and y-directions are taken for leg- and rung-direction, respectively. The potential strength is displayed in the perpendicular direction to the x-y plane. (c) A typical density distribution of 60 fermions with $U/t = 10$ and $V/t = 0.07$. The Mott core is formed in the center of the system. The atom density $(n_\uparrow + n_\downarrow)$ profile does not depend on the spin imbalance ratio p. (d) The Mott core region $\mathcal{M}$, where the density $(n_\uparrow + n_\downarrow)$ is a unit.

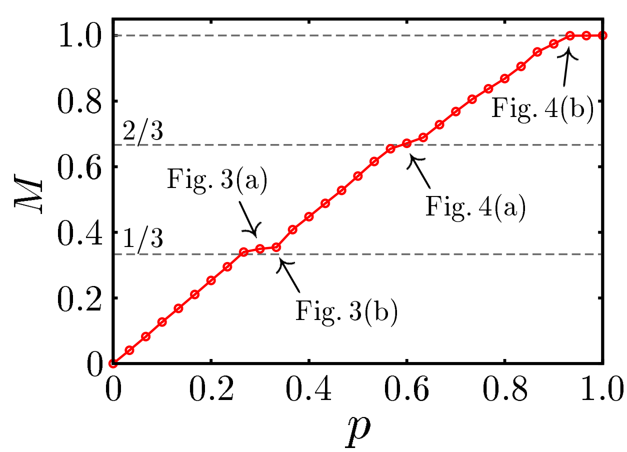

FIG. 2: The Mott core polarization (M) curve as a function of the population imbalance (p). The spin distributions at the values of p pointed by arrows are given in Figures 3 and 4.

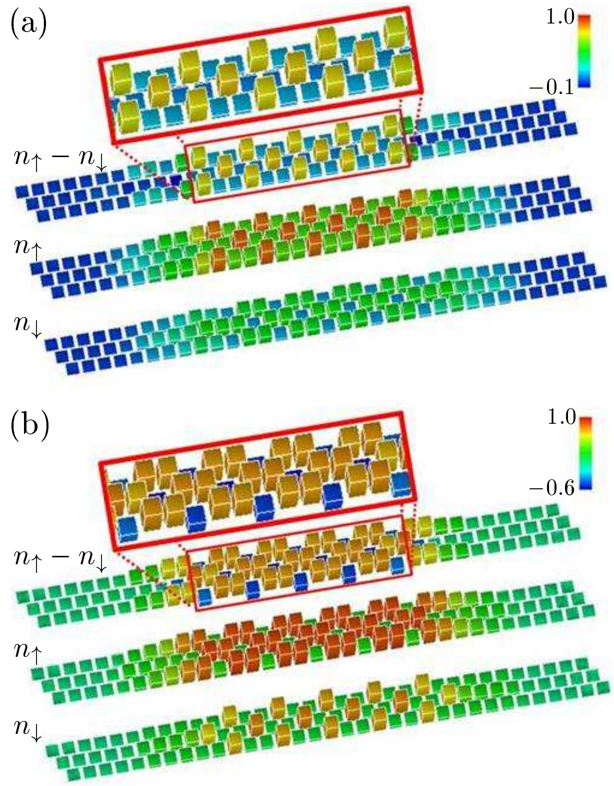

FIG. 3: Polarization ($n_\uparrow - n_\downarrow$), up-spin ($n_\uparrow$), and down-spin ($n_\downarrow$) density distributions at (a) $p = 0.30$ and (b) $p = 0.33$. The zoomed-in view is the spin distribution on the Mott core.

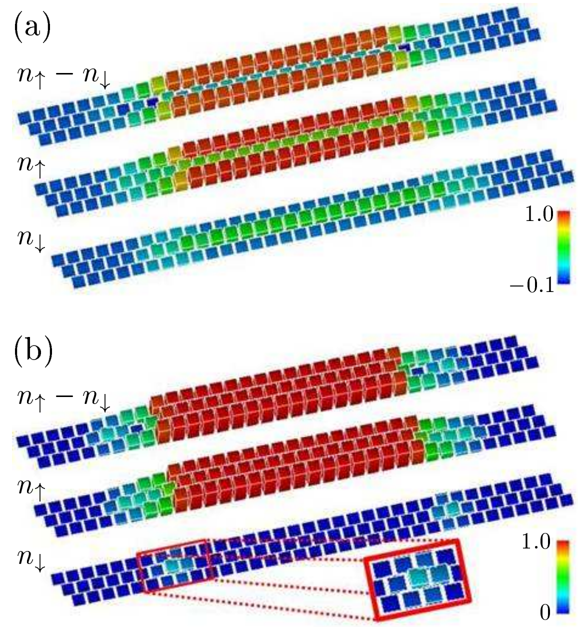

FIG. 4: Polarization ($n_\uparrow - n_\downarrow$), up-spin ($n_\uparrow$), and down-spin ($n_\downarrow$) density distributions at (a) $p = 0.60$ and (b) $p = 0.93$. The zoomed-in view is the down-spin distribution beside the Mott core.

confirmation in the triangular trapped FGOL.

The next is 2/3 kink observed at $p = 0.6$. Figure 4(a) shows the spin distribution pattern at $p = 0.6$. Its key feature is that up-spin and down-spin species are separately located. The down-spin species almost assemble along the central line. This brings about an extremely unbalanced but ordered profiles of the spin density as shown in Fig. 4(a).

The final is the fully-magnetized plateau region before the saturation imbalance. Figure 4(b) is a typical pattern on the final plateau. The majority, i.e., up-spin species fully dominate over the Mott core, while the minority is completely excluded outside the Mott core. This result also indicates that "environment" aids the full polarization of the Mott core. Such a perfect separation is quite easy to measure directly. We note that this full separation in addition to the plateaus is never observed in 3-leg square lattices.

Here, let us discuss why such non-trivial p-dependent profiles appear in the present system. The profile at 1/3 plateau is explained as follows. According to the OYA quantization condition, one then expects that the unit cell of the ground state are given in multiple of 3 sites at the imbalance range. The profiles observed as Fig. 3(a) and 3(b) are consistent with this OYA prediction and the well-known *up-up-down* structure [8, 9, 10, 11, 12, 13, 17] in 1/3 plateau. The possible profiles and the unit cells with *up-up-down* structure are three patterns given by the right and the left hand panels in Fig. 5(a) and one of Fig. 5(b). Among them, a consideration about x- and y-axis inversion symmetries in the present system leads to a superposition of two profiles as Fig. 5(a) or a profile as Fig. 5(b). In fact, the superposition state as Fig. 5(a) is observed in Fig. 3(a), and the state as Fig. 5(b) coincides with Fig. 3(b). In addition, the spin density profile in the 2/3 polarization kink is relevant to the OYA condition in 2/3 magnetization plateau. Although observed spin structure in Fig. 4(a) indicates that the unit cell of the ground state contains 3 sites, the structure is reconstructed by a superposition of two states with 6-sites periodicity predicted by the OYA condition as shown in Fig. 5(c). This implies that the observed kink is a shrunk piece of the 2/3 plateau. The idea can be confirmed by examining a situation mapped onto the XXZ model exhibiting 2/3 plateau [9]. The XXZ model is easily accessible through a setup of the spin-dependent hopping [23].

In conclusion, we explored whether or not FGOL is a good stage to study frustrated quantum spin systems. By applying DMRG method on 3-leg triangular Hubbard ladder with harmonic potential, we successfully found 1/3 plateau, 2/3 kink, and full polarization plateau in the Mott core polarization as a function of the population imbalance. In these states, we observed that the surrounded metallic region flexibly adjusts the spin imbalance to keep the plateau states on the Mott core. Especially, the minority is completely expelled from the Mott core in the full polarization plateau. In addition,

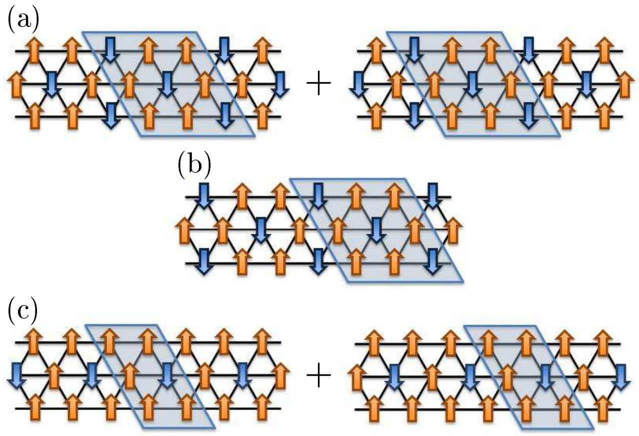

FIG. 5: Schematic figures of spin configurations to interpret the spin distribution of (a) Fig. 3(a), (b) Fig. 3(b), and (c) Fig. 4(a). The shaded area in each figure indicates the unit cell if the system is periodic.

we point out that the observed states in 1/3 plateau and 2/3 kink are consistent with the OYA predictions. We mention that FGOL can provide a new pathway to study the quantum frustrated systems. In addition, we point out that the excellent controllability is of a great advantage to quantum information processing using quantum frustrated systems [26].

Two of authors (M.O. and M.M.) wish to thank K. Okunishi and H. Onishi for illuminating discussion. The work was partially supported by Grant-in-Aid for Scientific Research on Priority Area "Physics of new quantum phases in superclean materials" (Grant Nos. 20029019 and 20029020) and for Scientific Research (Grant Nos. 20500044 and 20340096) from the Ministry of Education, Culture, Sports, Science and Technology of Japan. One of authors (M.M.) is supported by JSPS Core-to-Core Program-Strategic Research Networks, "Nanoscience and Engineering in Superconductivity".

[1] P.W. Anderson, Mater. Res. Soc. Bull. **8**, 153 (1973); P. Fazekas and P.W. Anderson, Philos. Mag. **30**, 423 (1974).
[2] R.R.P. Singh and D.A. Huse, Phys. Rev. Lett. **68**, 1766 (1992); B. Bernu, P. Lecheminant, C. Lhuillier, and L. Pierre, Phys. Rev. B **50**, 11048 (1994); L. Capriotti, A.E. Trumper, and S. Sorella, Phys. Rev. Lett. **82**, 3899 (1999); D.J.J. Farnell, R.F. Bishop and K.A. Gernoth, Phys. Rev B **63**, 220402(R) (2001).
[3] M. Siqueira, J. Nyéki, B. Cowan, and J. Saunders, Phys. Rev. Lett. **76**, 1884 (1996); K. Ishida, M. Morishima, K. Yawata, and H. Fukuyama, *ibid* **79**, 3451 (1997); A. Casey, H. Patel, J. Patel, J. Nyéki, B.P. Cowan, and J. Saunders, J. Low Temp. Phys. **113**, 293 (1998); E. Collin, S. Triqueneaux, R. Harakaly, M. Roger, C. Bäuerle, Yu.M. Bunkov, and H. Godfrin, Phys. Rev. Lett. **86**, 2447 (2001); R. Masutomi, Y. Karaki, and H. Ishimoto, *ibid* **92**, 025301 (2004).
[4] T. Itou, A. Oyamada, S. Maegawa, M. Tamura, and R. Kato, Phys. Rev. B **77**, 104413 (2008).
[5] Y. Shimizu, K. Miyagawa, K. Kanoda, M. Maesato, and G. Saito, Phys. Rev. Lett. **91**, 107001 (2003); Y. Kurosaki, Y. Shimizu, K. Miyagawa, K. Kanoda, and G. Saito, *ibid* **95**, 177001 (2005); Y. Shimizu, K. Miyagawa, K. Kanoda, M. Maesato, and G. Saito, Phys. Rev. B **73**, 140407(R) (2006).
[6] S. Yamashita, Y. Nakazawa, M. Oguni, Y. Oshima, H. Nojiri, Y. Shimizu, K. Miyagawa, and K. Kanoda, Nature Phys. **4**, 459 (2008).
[7] M. Yamashita, N. Nakata, Y. Kasahara, T. Sasaki, N. Yoneyama, N. Kobayashi, S. Fujimoto, T. Shibauchi, and Y. Matsuda, Nature Phys. **5**, 44 (2009).
[8] S. Miyashita, J. Phys. Soc. Jpn. **55**, 3605 (1986).
[9] H. Nishimori and S. Miyashita, J. Phys. Soc. Jpn. **55**, 4448 (1986).
[10] D.C. Cabra, A. Honecker, and P. Pujol, Phys. Rev. Lett. **79**, 5126 (1997); Phys. Rev. B **58**, 6241 (1998); A. Honecker, J. Phys.: Condens. Matter **11**, 4697 (1999); A. Honecker, J. Schulenburg, and J. Richter, J. Phys.: Condens. Matter **16**, S749 (2004).
[11] K. Okunishi and T. Tonegawa, J. Phys. Soc. Jpn. **72**, 479 (2003).
[12] S. Miyahara, K. Ogino, and N. Furukawa, Physica B **378–380**, 587 (2006).
[13] T. Ono, H. Tanaka, H.A. Katori, F. Ishikawa, H. Mitamura, and T. Goto, Phys. Rev. B **67**, 104431 (2003); T. Ono, H. Tanaka, O. Kolomiyets, H. Mitamura, T. Goto, K. Nakajima, A. Oosawa, Y. Koike, K. Kakurai, J. Klenke, P. Smeibidle, and M. Meißner, J. Phys.: Condens. Matter **16**, S773 (2004); T. Ono, H. Tanaka, O. Kolomiyets, H. Mitamura, F. Ishikawa, T. Goto, K. Nakajima, A. Oosawa, Y. Koike, K. Kakurai, J. Klenke, P. Smeibidle, M. Meißner, R. Coldea, A.D. Tennant, and J. Ollivier, Prog. Theor. Phys. Suppl. **159**, 217 (2005); H. Tsujii, C.R. Rotundu, T. Ono, H. Tanaka, B. Andraka, K. Ingersent, and Y. Takano, Phys. Rev. B **76**, 060406(R) (2007).
[14] M. Oshikawa, M. Yamanaka, and I. Affleck, Phys. Rev. Lett. **78**, 1984 (1997); M. Oshikawa, *ibid* **84**, 1535 (2000).
[15] S. R. White, Phys. Rev. Lett. **69**, 2863 (1992); Phys. Rev. B **48**, 10345 (1993).
[16] For recent reviews, see e.g., U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005); K. A. Hallberg, Adv. Phys. **55**, 477 (2006), and references therein.
[17] J. Alicea, A.V. Chubukov, and O.A. Starykh, arXiv:0809.3800.
[18] See for reviews, e.g., I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008); M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen(De), and U. Sen, Adv. Phys. **56**, 243 (2007), and references therein.
[19] M. Rigol and A. Muramatsu, Phys. Rev. A **69**, 053612 (2004).
[20] M. Machida, S. Yamada, Y. Ohashi, and H. Matsumoto, Phys. Rev. Lett. **93**, 200402 (2004).
[21] R. Jördens, N. Strohmaier, K. Günter, H. Moritz, and T. Esslinger, Nature **455**, 204 (2008).
[22] U. Schneider, L. Hackermüller, S. Will, Th. Best,

I. Bloch, T.A. Costi, R.W. Helmes, D. Rasch, and A. Rosch, Science **322**, 1520 (2008).
[23] L.-M. Duan, E. Demler, and M.D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
[24] M. Machida, M. Okumura, S. Yamada, T. Deguchi, Y. Ohashi, and H. Matsumoto, Phys. Rev. B **78**, 235117 (2008).
[25] S. Yamada, M. Okumura, and M. Machida, arXiv:0707.0159.
[26] J.K. Pachos and M.B. Plenio, Phys. Rev. Lett. **93**, 056402 (2004); Phys. Rev. A **70**, 053620 (2004).